Soret-induced discrepancies between pure fluid and binary liquid steady convection

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Abstract

We investigate the various 2D convective states of both pure and binary liquids enclosed in a vertical cylinder (of equal height and diameter) which is heated from below. Solutions are sought by direct numerical simulation of the set of balance equations ruling such systems. A detailed analysis of the differences that arise between pure and binary liquid steady flows reveals that the observed discrepancies can be tracked down to the existence of a small roll located in the 'corner' of the enclosure. The growth of this local region, which leads to the destabilization of the steady convective states, is moreover shown to be ruled by the Soret effect.

1 Background

Fluid motion driven by thermal gradients is a common feature of many natural and industrial systems. The traditional problem of a mono-component fluid layer heated from below is a paradigm of the rich spatiotemporal behaviors that can arise in non-linear systems driven away from equilibrium. Since the governing equations are well known and the experimental setup is sufficiently simple to allow controlled experiments, it has become the context of many studies (see for instance [1] for a review of recent developments) on pattern formation and related topics. The 'simplicity' of this system comes from the fact that only two parameters are needed to describe it: the Prandtl Pr and Rayleigh Ra numbers. The first solely depends on the characteristics of the fluid (and is thus thereafter fixed at a given value) whereas the second is proportional to the temperature difference applied to the layer, which makes it a perfect control parameter. The typical evolution of the system with increasing Ra is the following: The layer first remains motionless for too small an applied temperature difference. As the later is increased, it

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eventually exceeds a critical value (Ra_c) and steady convection sets in. More complicated dynamics, such as time-dependent convection arise as the system is taken further away from onset.

In two-component miscible liquid mixtures, mass fraction and temperature gradients are coupled by the Soret effect. According to the sign thereof (or equivalently of the non-dimensional parameter it enters in, the separation ratio Ψ), the solute (which we take to be the heaviest of the two components) will migrate towards either the warmer ($\Psi < 0$) or colder ($\Psi > 0$) part of the container. The resulting Soret-driven mass fraction gradient will thus, under normal gravity conditions, induces a solutal buoyancy that will strengthen (when $\Psi > 0$) or oppose ($\Psi < 0$) the thermal one, consequently rushing or delaying the onset of convection. When thermal and solutal buoyancies compete, the interplay between the two typically leads to finite amplitude oscillatory convective regimes that arise as soon as the quiescent state turns unstable. These periodic solutions show rich and complex spatiotemporal behaviors that have been intensively studied over the past decades or so (see for instance [4] for a recent review).

Oscillatory convection is usually found to last over a given range in Ra beyond which a transition to steady overturning convection (SOC) occurs. These steady states are often found to be very similar to those obtained in pure fluids ($\Psi = 0$) under the same conditions. This feature is generally evoked (see for instance [2, 4], among many others) as resulting from the efficient mixing process of strong convective motion (which is, loosely put, proportional to Ra). Using the assumption that binary liquid convective states might be seen as a perturbation of pure fluid ones, a theoretical model predicting the transition between oscillatory and steady convection has even be derived in [2]. Experiments [5] conducted to verify the proposed model have however invalidated it (or at the very least shown that its range of application is quite limited). Nevertheless, studies relating investigations on the steady convective states of binary liquids (with $\Psi < 0$) are rather rare. A very instructive exception is however given in [3] where experimental results unambiguously show that the stationary convective patterns that develop in a binary liquid are unlike those obtained in a pure fluid under the same conditions.

The present contribution tackles this last topic in that we seek to understand to what extent pure and binary liquid steady states can be alike and how discrepancies that arise can be related to the Soret effect.

2 Geometry and parameters

We consider a vertical cylinder (of height h and radius $h/2$) heated from below: both top and bottom plates are maintained at given temperatures. The circumference of the cylinder is taken to be perfectly insulated. Realistic no-slip and impermeable conditions are imposed on all the boundaries of the enclosure.
The two parameters required to describe the pure fluid system are:

- The Rayleigh number $Ra = \frac{\alpha \Delta T g h^3}{\kappa \nu}$, where h is the height of the cylinder, g the gravitational acceleration, α the thermal expansion coefficient, κ and ν the thermal and momentum diffusivities and ΔT the imposed temperature difference between bottom and top plates.

- The Prandtl number $Pr = \frac{\nu}{\kappa}$.

When a binary liquid is considered, two additional parameters are required:

- The Lewis number $Le = \frac{\kappa_s}{\kappa}$, where κ_s is the solute diffusivity.

- The separation ratio $\Psi = \frac{\beta}{\alpha} S_T C_0 (1 - C_0)$, where C_0, β, and S_T stand for mean mass fraction, mass expansion and Soret coefficients.

In what follows, the values of these last three parameters are set to $Pr = 1$, $Le = 10^{-1}$ and $\Psi = -0.2$.

3 Main results

The chosen geometrical configuration turns out to yield multiple branches of solutions (as displayed by the bifurcation diagram of figure 1), consisting not only of both stationary and periodic states, but also based on two distinct flow patterns: In the first case, flows mainly consist of a main roll that fills the whole enclosure while the spatial structure of the second is based on a pair of (vertically) stacked rolls. There is much to be said on all these solutions and their features. We will, in the present extended abstract, leave many aside and focus on the evolution (and destabilization) of the steady one roll patterns (upper curves of the bifurcation diagram).

In the pure fluid case, the steady states branch stems from the quiescent state (at $Ra_c = 10870$) and extends up to $Ra = 84903$. As for the binary liquid branch, it begins at $Ra = 14938$ (at finite amplitude) and extends up to $Ra = 107700$. In both cases, at the beginning of the branch flows consist

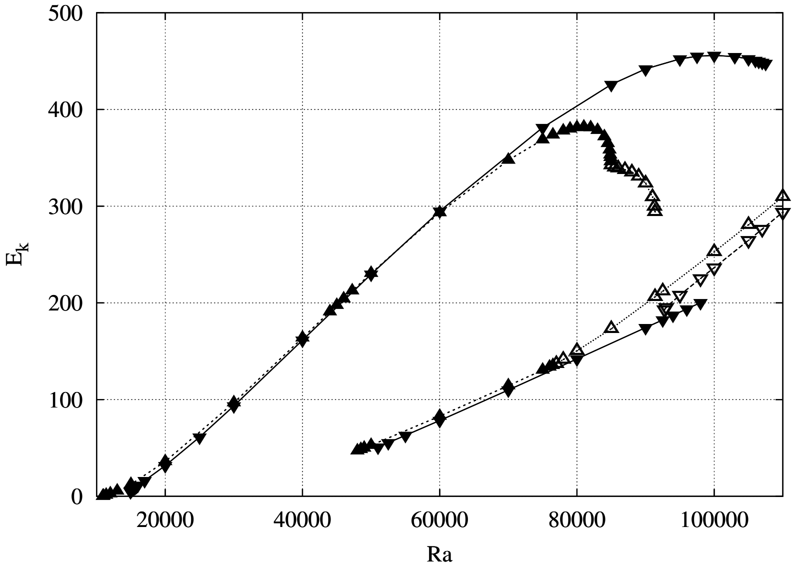

Figure 1: Global kinetic energy of both pure fluid (upward triangles) and binary liquid (downward triangles) convective states in the NS configuration. Filled (empty) symbols stand for steady (oscillatory) states. The upper curves correspond to single roll flows and the lower ones to stacked rolls states.

of a single large roll that fills the whole of the container. There are however much smaller rolls located in the 'corners' (where horizontal and side walls meet) of the cylinder. These tiny corner rolls are found to slowly grow with increasing Ra values. As displayed in figure 1, pure and binary liquid branches collapse over $Ra \in [30000, 70000]$. In this range, velocity and temperature distributions in both systems are next to identical. At higher Ra the branches again part. This discrepancy is in fact due to distinct growth rates of the corner roll which is reduced in the binary liquid case, comparatively to the pure fluid one. Apart from this feature, the ultimate convective regimes (i.e: those prior to the termination of the branch) are of similar structure (displayed, for the binary liquid case, in figure 2).

The interesting feature depicted by the fields displayed in figure 2 (which is also observed in the pure fluid case) is that the corner roll develops in an area where temperature and mass fraction gradients are mainly vertical. Rather surprisingly, these gradients are moreover found to barely evolve in the range in Ra over which a significant growth of the corner roll is observed.

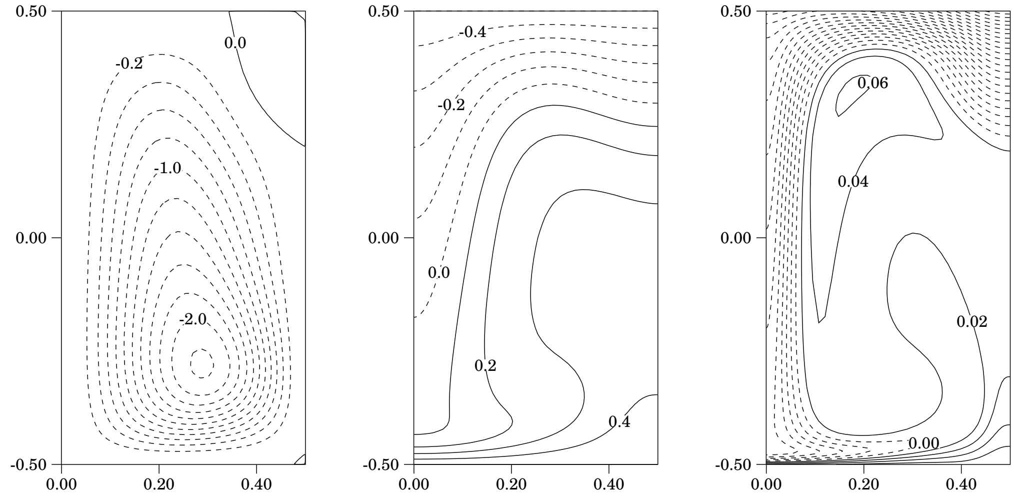

Figure 2: Streamfunction, temperature and mass fraction contours of the $Ra = 1.075\,10^5$ binary liquid steady state.

Taking into account this fact and noting that velocities in the corner roll are rather small (typically less than twenty times those found in the main roll), it is reasonable to consider the corner roll as a locally quasi-quiescent region. A local Rayleigh number Ra_h based on the height h and the (constant) local buoyancy gradient could hence be derived. As the height h of the layer increases (consequently to a rise of Ra, as shown in figure 3), so does Ra_h, which implies that the greater the extension of the quasi-quiescent region is, the less stable it will be. Eventually, h becomes such that the quasi-quiescent region turns unstable (Ra_h reaches a critical value) leading to the growth of the corner cell that destabilizes the whole steady state.

The physical mechanism responsible for the termination of the steady states branch is therefore the same than the one leading to the destabilization of the quiescent state. The well known consequences (stabilizing if $\Psi < 0$ and destabilizing for $\Psi > 0$) induced by the presence of the Soret effect on the latter hence also apply to the former.

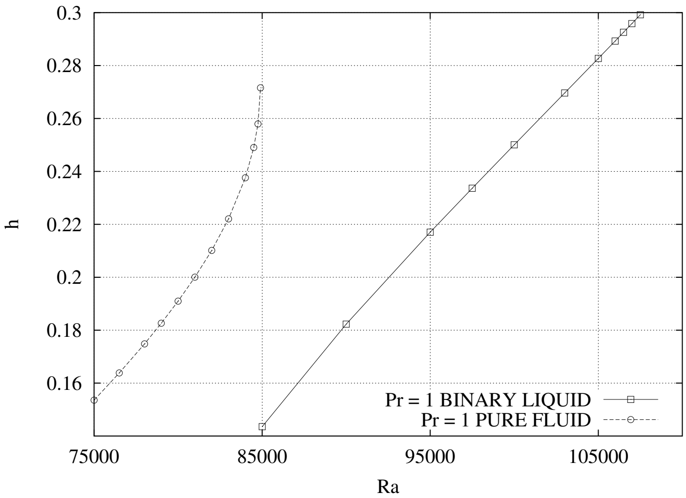

Figure 3: Height of the corner roll, for both pure fluid and binary liquid steady states.

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